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**Intermountain
Forest and Range
Experiment Station**

Research Paper
INT-283

August 1981

Crown Width and Foliage Weight of Northern Rocky Mountain Conifers

Melinda Moeur

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THE AUTHOR

MELINDA MOEUR received her master's degree in forestry from the University of Minnesota in 1980. As a research forester at the Forestry Sciences Laboratory, Moscow, Idaho, she has worked on linking vegetation, watershed, and wildlife system models into a decision-support system.

RESEARCH SUMMARY

Equations were derived for predicting crown width of trees from diameter, height, crown length, and basal area per acre, and for predicting foliage weight of trees from diameter, height, crown length, age, relative diameter, and number of trees per acre. Coefficients were estimated for 11 conifer species in northern Idaho and western Montana. Embedding these equations in the prognosis model for stand development will enhance the prediction of vegetation characteristics needed for interfacing insect outbreak, wildlife habitat, and watershed models.

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INTRODUCTION

This paper reports on the development of multiple linear regression models predicting individual tree crown width and foliage weight for 11 conifer species in northern Idaho and western Montana. The equations have been applied in a spruce budworm version[1] of the stand prognosis model (Stage 1973) being currently developed. The crown width and foliage biomass equations, when linked with other relationships in the spruce budworm system, help predict insect damage and dispersal of larvae during simulated outbreaks. The crown width function has also been used to predict percent stand crown cover in the prognosis model (Stage 1973) for linkage with proposed watershed[2] and big game habitat prediction[3] submodels.

The relationship between crown width and tree diameter has been of interest to several researchers in developing crown competition indices (see for example Krajicek and others 1961 or Vezina 1963). This relationship has been expressed as:

$$\text{crown width} = a + b\,(\text{d.b.h.})$$

In some studies, for example Stiell (1966), stand density inversely influenced tree crown width. Vezina (1964) used stand basal area to predict stand crown closure. Bonnor (1964), in studying the relationship between diameter at breast height (d.b.h.) and crown width on vertical photography, found no effect from stand density. The current study investigates the contribution of a number of stand and individual tree characteristics in predicting crown width by species.

The second part of this study looks at the relationship between individual tree foliage weight and measured tree and stand characteristics. Kittredge (1944) developed the following logarithmic expression to estimate leaf weight for several conifer and hardwood species:

$$\ln\,(\text{leaf weight}) = b\,(\ln \text{d.b.h.}) - a$$

He noted conflicting site and age effects, possibly confounded by geographical location. The effects of density and crown class as sources of variation were not separated out in his study.

Brown (1978) approached the problem of estimating foliage weight indirectly by first developing total crown weight as a function of tree diameter, height, and crown length, and then estimating the proportion of the total crown in foliage in a separate equation. In addition, Brown presented more than one model form for each species depending on crown class and size class. The current study used Brown's data with the purpose of simplifying the direct estimation of foliage weight by investigating one or two model forms applicable to all tree species considered.

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[1] Crookston, N.L. The interface between the stand prognosis model and the western spruce budworm model — Version 1.2. Unpubl. rep. on file, Intermt. For. and Range Exp. Stn., Forestry Sciences Laboratory, Moscow, Idaho, October 1980, 30 p.

[2] Simons, D.B., R.M. Li, and T.J. Ward. 1980. Development of a generalized planning model for assessing effects of land use changes on watershed responses and aquatic habitat conditions. Unpubl. study plan, 14 p. Colo. State Univ. Dep. Civil Eng., Fort Collins.

[3] Peek, J.M. 1970. Study plan: evaluations of elk habitat use patterns and relationships in the Gospel-Hump area of central Idaho, 10 p. Univ. Idaho, Wildl. Resour., Dep. For., Wildl. and Range Sci., Moscow.

DATA

The data studied are a subsample of trees used by Brown (1978) in estimating crown weights. Trees deleted from the original data set were those lacking measurements for crown width or foliage weight. Refer to Brown (1978) for a more complete description and listing of the data.

Measurements from dominant and codominant trees were studied for the following 11 species (abbreviations are used in following tables and figures):

DF	Douglas-fir	*Pseudotsuga menziesii* (Mirb.) Franco
ES	Engelmann spruce	*Picea engelmannii* Parry
GF	Grand fir	*Abies grandis* (Dougl.) Lindl.
LP	Lodgepole pine	*Pinus contorta* Dougl.
PP	Ponderosa pine	*Pinus ponderosa* Laws.
AF	Subalpine fir	*Abies lasiocarpa* (Hook.) Nutt.
WH	Western hemlock	*Tsuga heterophylla* (Raf.) Sarg.
WL	Western larch	*Larix occidentalis* Nutt.
WC	Western redcedar	*Thuja plicata* Donn
WP	Western white pine	*Pinus monticola* Dougl.
BP	Whitebark pine	*Pinus albicaulis* Engelm.

In addition, trees of intermediate and suppressed crown classes were available only for western redcedar (WC-INT), ponderosa pine (PP-INT), grand fir (GF-INT), and Douglas-fir (DF-INT). Tables 1 and 2 show the distribution of data used in developing the models.

PROCEDURE

The models were developed by means of a regression screening method. First, the relationships between the two dependent variables of crown width and foliage weight, and stand and tree characteristics were studied by screening for the best combinations of the independent variables and their logarithmic transformations. Variable screening and coefficient estimation were done in a multiple linear regression computer program, REX (Grosenbaugh 1967). The following relationships between the dependent and independent variables were examined:

$$Y = f(D, D^2, H, CL, CR, A, TPA, BA, DREL, SP, SI)$$

$$\ln Y = f(\ln D, (\ln D)^2, \ln H, \ln CL, \ln CR, \ln A, \ln TPA, \ln BA, \ln DREL, SP, SI)$$

Table 1.—Distribution of sample trees by species and crown class used in estimating crown width

	Number of trees		Range of Measurements			
Species[1]	<3.5 inches	≥3.5 d.b.h.	d.b.h.	Height	Crown length	Stand basal area
			Inches	*Feet*		*Feet^2/acre*
BP	12	7	0 - 10.6	3 - 44	2 - 36	1 - 181
WP	17	8	0 - 24.7	3 - 126	3 - 88	1 - 299
WC	14	14	0 - 26.6	2 - 127	2 - 98	1 - 300
WC-INT	11	11	0 - 10.6	4 - 64	2 - 42	45 - 335
WL	15	9	0 - 21.8	3 - 129	3 - 75	1 - 271
WH	16	12	0 - 21.4	4 - 126	3 - 111	1 - 320
AF	15	10	0 - 12.7	3 - 93	2 - 70	1 - 260
PP	15	20	0 - 34.0	2 - 144	2 - 89	1 - 325
PP-INT	12	9	0 - 12.1	4 - 70	1 - 44	22 - 280
LP	12	7	0 - 15.6	2 - 84	1 - 60	1 - 271
GF	16	13	0 - 20.4	3 - 137	3 - 108	1 - 432
GF-INT	13	9	0 - 11.8	4 - 72	2 - 60	39 - 296
ES	14	8	0 - 23.2	2 - 128	2 - 105	1 - 240
DF	16	13	0 - 33.9	4 - 143	4 - 86	2 - 238
DF-INT	12	10	0 - 11.2	4 - 83	1 - 47	36 - 360
Total	210	160				

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[1] See text page 2 for species list.

Table 2.—Distribution of sample trees by species and crown class used in estimating foliage weight

Species[1]	Number of trees <3.5 inches d.b.h.	Number of trees ≥3.5 inches d.b.h.	Range of measurements: d.b.h.	Height	Crown length	Number of trees per acre	Age
			Inches	-------------*Feet*	------------		*Years*
BP	12	4	0 - 7.4	3 - 38	2 - 27	461 - 6324	6 - 182
WP	17	2	0 - 7.4	3 - 66	3 - 46	300 - 4500	4 - 55
WC	14	11	0 - 26.6	2 - 127	2 - 98	110 - 7800	3 - 237
WC-INT	11	11	0 - 10.6	4 - 64	2 - 42	169 - 3611	16 - 162
WL	15	3	0 - 6.6	3 - 50	3 - 29	600 - 5100	3 - 55
WH	16	4	0 - 7.0	4 - 38	3 - 38	848 - 7200	4 - 32
AF	15	10	0 - 12.7	3 - 93	2 - 70	87 - 15121	4 - 201
PP	15	20	0 - 34.0	2 - 144	2 - 89	5 - 7200	6 - 217
PP-INT	12	9	0 - 12.1	4 - 70	1 - 44	142 - 3344	11 - 140
LP	12	0	0 - 1.8	2 - 13	1 - 12	300 - 6000	5 - 11
GF	16	8	0 - 15.6	3 - 79	3 - 70	300 - 7079	4 - 124
GF-INT	13	9	0 - 11.8	4 - 72	2 - 60	359 - 5804	15 - 117
ES	14	3	0 - 9.0	2 - 57	2 - 44	600 - 12730	7 - 153
DF	16	10	0 - 33.9	4 - 143	4 - 86	21 - 7200	6 - 262
DF-INT	12	10	0 - 11.2	4 - 83	1 - 47	261 - 4500	12 - 145
Total	210	114					

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[1] See text page 2 for species list.

where:

D = diameter breast height, inches (cm)
H = height, ft (m)
CL = crown length, ft (m)
CR = crown ratio
A = tree age, years
TPA = number of trees per acre (number per hectare)
BA = stand basal area, ft^2/ac (m^2/ha)
DREL = relative diameter (tree d.b.h/quadratic mean stand diameter)

$$SP_i = \begin{cases} 1 \text{ if tree is of species i,} \\ 0 \text{ otherwise} \end{cases}$$

$$SI_j = \begin{cases} 1 \text{ if tree is in site class j,} \\ 0 \text{ otherwise} \end{cases}$$

Y = predicted crown width (*CW*), ft (m) or foliage weight (*WT*), lbs (kg)

RESULTS

Variable selection began with the model chosen in the REX screening process as having the lowest mean square error/var(Y). A general model was chosen when coefficients that were not significantly different from zero when tested by the T-statistic at the 0.05 level, were deleted from the parameter set, and all remaining coefficients were significant when the model was refit.

Refinements on the general model were investigated by allowing each parameter to vary by species. Species x parameter interactions that significantly improved the regression sum of squares were then included in the final model form.

In developing the equations, the data were divided into two sets consisting of those trees 3.5 inches (8.9 cm) d.b.h. and larger, and trees less than 3.5 inches (8.9 cm). Each set was then modeled independently. The break in diameter was chosen to correspond to prognosis model methodology that predicts diameter growth for large trees and height growth for smaller trees. For the set of smaller trees, d.b.h. was not included in the list of possible predictor variables. The fundamental reason for modeling the two size classes of trees differently is that for small trees, either diameter at breast height is not available, or is not as meaningful a dimension as it is for larger trees.

Crown Width

For 160 trees 3.5 inches (8.9 cm) d.b.h. and larger, the general model is:

$$\ln(CW) = b_0 + b_1\ln(D) + b_2\ln(H) + b_3\ln(CL)$$

In this and all subsequent models, a log-log model form was chosen in order to linearize the geometric relationships and stabilize the variance structure commonly encountered when working with a wide range of data (for instance, when variance is proportional to tree size). In this and subsequent models, the second assumption is borne out by the uniform random patterns shown in scatterplots of residuals versus predicted values in the log scale.

It is interesting to note that in this model no stand density term is included as a measure of competition. However, if the log model were inversely transformed to the natural scale, the combination of the *CL* term with its positive coefficient, and *H* with its negative coefficient (table 3) could be interpreted as crown ratio. The effect of competition on predicted crown width would be represented by crown ratio instead of stand density. It is assumed that this effect would be positive, that is, crown width would increase with increasing crown ratio.

Table 4 is an analysis of variance showing the improvement in the regression sum of squares when variables in the general model are allowed to vary by species. For example, varying the intercept term by species improves the regression by a significant amount. Then, when either ln(*D*) or ln(*H*) are allowed to vary by species, the fit is further significantly improved. Of these two alternatives, the latter has a slight advantage. Although fitting ln(*CL*) by species, or any more successively complex model also improves the regression, the signs of the coefficients for the *H* and *CL* terms become inconsistent (some species with positive coefficients and some with negative coefficients for a single parameter). The inconsistencies of sign among species could not be logically explained, for example, as a pattern indicating ranking of species by tolerance. Therefore it is assumed that the terms change sign because the model is overfitting the particular data set. Acceptable models were thus constrained to those with consistent coefficients. The final model chosen is that in which the intercept and height terms vary by species (indicated by the subscript *i* on the parameter):

$$\ln(CW) = b_{0_i} + b_1\ln(D) + b_{2_i}\ln(H) + b_3\ln(CL)$$

The coefficients for this model, shown in table 3, gave a mean square error of 0.04898. The equation overpredicts crown width in the natural scale for four species and underpredicts for seven species, as shown in table 5. The mean crown width is underestimated by 5.7 ft (1.7 m).

Negative bias (mean residual deviation from zero), introduced when the inverse logarithmic transformation is used to convert log-normally distributed estimates to the original natural scale, can be approximately corrected by adding one-half the residual variance to the estimate on the log scale (Baskerville 1972). This amounts to multiplying the estimate of crown width in the natural scale by exp[½MSE]. Thus a factor of $e^{1/2(.04898)} =$ 1.025 may be applied to the estimate in the natural scale to correct for underprediction. After "bias adjustment," mean crown width is overestimated slightly, 1.2 ft (0.4 m).

Table 3.—Coefficients for estimating ln (crown width) of trees 3.5 inches d.b.h. and larger:
$\ln(CW) = b_{0_i} + b_1\ln(D) + b_{2_i}\ln(H) + b_3\ln(CL)$

	Variable coefficients	
Species[1]	Intercept b_{0_i}	ln(H) b_{2_i}
BP	−0.91984	−0.07299
WP	4.30800	−1.37265
WC	2.79784	−.89666
WL	2.31359	−.80919
WH	1.32772	−.52554
AF	1.74558	−.73972
LP	1.06804	−.55987
GF	2.20611	−.76936
ES	3.76535	−1.18257
DF	3.02271	−1.00486
PP	1.62365	−.68098
Variables	Variable coefficients	
ln(*D*)	$b_1 = 1.08137$	
ln(*CL*)	$b_3 = 0.29786$	

Regression sum of squares/d.f. = 33.37909/23
Error sum of squares/d.f. = 6.66189/136
Mean square error = 0.04898
R^2 = 0.8336

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[1] See text page 2 for species list.

Table 4.—Analysis of improvement of ln(crown width)[1] regression models attainable by varying parameters by species, trees 3.5 inches d.b.h. and larger

Model[2]	Source	Remarks	Degrees of freedom	Marginal sum of squares	Mean square reduction	F[3]
(1) $b_0 + b_1\ln(D) + b_2\ln(H) + b_3\ln(CL)$	Reduction due to model (1)		3	24.09107	8.03036	206.65
(2) $\underline{b_0} + b_1\ln(D) + b_2\ln(H) + b_3\ln(CL)$	Reduction due to model (2) -reduction due to model (1)		10	7.46691	.74669	19.21
(3) $\underline{b_0} + \underline{b_1}\ln(D) + b_2\ln(H) + b_3\ln(CL)$	Reduction due to model (3) -reduction due to model (2)		10	1.45889	.14589	3.75
(4) $\underline{b_0} + b_1\ln(D) + \underline{b_2}\ln(H) + b_3\ln(CL)$	Reduction due to model (4) -reduction due to model (2)		10	1.82111	.18211	4.69
(5) $\underline{b_0} + b_1\ln(D) + b_2\ln(H) + \underline{b_3}\ln(CL)$	Reduction due to model (5) -reduction due to model (2)	inconsistent signs in *CL* terms	10	2.34274	.24327	6.03
(6) $\underline{b_0} + \underline{b_1}\ln(D) + \underline{b_2}\ln(H) + b_3\ln(CL)$	Reduction due to model (6) -reduction due to model (4)	inconsistent signs in *H* terms	10	1.10706	.11071	2.85
(7) $\underline{b_0} + \underline{b_1}\ln(D) + b_2\ln(H) + \underline{b_3}\ln(CL)$	Reduction due to model (7) -reduction due to model (3)	inconsistent signs in *CL* terms	10	1.36227	.13623	3.51
(8) $\underline{b_0} + b_1\ln(D) + \underline{b_2}\ln(H) + \underline{b_3}\ln(CL)$	Reduction due to model (8) -reduction due to model (4)	inconsistent signs in *H* and *CL* terms	10	1.35748	.13575	3.49
complete model $\underline{b_0} + \underline{b_1}\ln(D) + \underline{b_2}\ln(H) + \underline{b_3}\ln(CL)$	Error	inconsistent signs in *H* and *CL* terms	116	4.50798	.03886	

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[1] Mean $\ln(CW)$ = 2.53253 feet; standard deviation = 0.50183.

[2] Terms varying by species are underlined.

[3] Tabulated F values: $F^{10}_{116,0.01} = 2.49$.

Table 5.—Summary of residuals (natural scale) from prediction equations for crown width of trees. Bias adjustment factors are 1.025 for trees greater than or equal to 3.5 inches d.b.h., and 1.031 for trees less than 3.5 inches

Species[1]	No. of trees	Trees ≥3.5 inches d.b.h. Σ (obs-pred) without correction	Trees ≥3.5 inches d.b.h. Σ (obs-pred) with bias correction	No. of trees	Trees <3.5 inches d.b.h. Σ (obs-pred) without correction	Trees <3.5 inches d.b.h. Σ (obs-pred) with bias correction
BP	7	− 0.71	−1.93	12	0.61	−0.19
WP	8	1.59	− .36	17	− .03	−2.12
WC	25	−1.25	−11.26	25	1.70	−2.00
WL	9	− .03	−3.08	15	1.25	− .27
WH	12	.15	−5.66	16	2.08	.20
AF	10	− .80	−3.00	15	.58	−1.01
LP	7	5.68	3.73	12	.29	− .08
GF	22	1.80	−6.09	29	1.55	−2.40
ES	8	2.35	− .43	14	.52	− .98
DF	23	9.95	2.07	28	4.19	.78
PP	29	21.02	10.30	27	8.84	5.66
Weighted mean residual (ft/tree)		5.67	−1.18		2.48	−0.07
(m/tree)		(1.73)	(−0.36)		(0.76)	(−0.02)

[1] See text page 2 for species list.

The general model for prediction of crown width for 210 trees less than 3.5 inches (8.9 cm) d.b.h. is:

$$\ln(CW) = b_1\ln(H) + b_2\ln(CL) + b_3\ln(BA)$$

In this model for smaller trees the density effect, as measured by the basal area term, is positive rather than negative as might be expected (table 6). One possible explanation is that there is no competition effect among small trees due to stocking. Instead, the density effect is positive in that small trees in a well-stocked stand expand laterally to utilize all available space. As the stand matures with time, closure of the canopy begins, and individual tree crowns start to compete, stocking may negatively affect crown width. This argument would assume that the measured stands are mainly even-aged, and would not hold for suppressed trees with weak crowns. That more than three-fourths of the data are from dominant or codominant trees helps support this explanation.

Table 6.—Coefficients for estimating ln(crown width) of trees less than 3.5 inches: $\ln(CW) = b_{1_i}\ln(H) + b_2\ln(CL) + b_3\ln(BA)$

Species[1]	Variable coefficients ln(H) b_{1_i}
BP	0.07049
WP	.37031
WC	.46452
WL	.23846
WH	.25622
AF	.33722
LP	.26342
GF	.38503
ES	.33089
DF	.32874
PP	.36380
Variables	Variable coefficients
ln(CL)	$b_2 = .28283$
ln(BA)	$b_3 = .04032$

Regression sum of squares/d.f.	=	371.02441/13
Error sum of squares/d.f.	=	11.89093/197
Mean square error	=	0.06036
R^2	=	0.9689

[1] See text page 2 for species list.

Table 7 shows the effects on the general model of varying parameters by species. Since the intercept term in the original model was not significantly different from zero, all subsequent regressions were fitted through the origin. Varying $\ln(H)$, $\ln(CL)$, or $\ln(BA)$ by species significantly improves the fit. However, inconsistency in signs for the $\ln(CL)$ or $\ln(BA)$ terms occurs in all except model (2). Thus model (2) was chosen for predicting crown width for trees less than 3.5 inches (8.9 cm) d.b.h.:

$$\ln(CW) = b_{1_i}\ln(H) + b_2\ln(CL) + b_3\ln(BA)$$

where the subscript *i* denotes a different coefficient for the $\ln(H)$ term for every species.

The coefficients in table 6 gave a mean square error of 0.06036. The model underpredicts crown width by an average of 2.5 ft (0.8 m) (table 5). A factor of $e^{1/2(.06036)} = 1.031$ may be applied to the estimate in the natural scale to correct for negative bias. When this is done, crown width is overpredicted by only 0.07 ft (0.02 m).

Table 7.—Analysis of improvement of ln(crown width)[1] regression models attainable by varying parameters by species, trees less than 3.5 inches d.b.h.

	Model[2]	Source	Remarks	Degrees of freedom	Marginal sum of squares	Mean square reduction	F[3]
(1)	$b_1\ln(H) + b_2\ln(CL) + b_3\ln(BA)$	Reduction due to model (1)		3	364.13574	121.37858	2063.
(2)	$\underline{b_1}\ln(H) + b_2\ln(CL) + b_3\ln(BA)$	Reduction due to model (2) -reduction due to model (1)		10	6.88867	.68887	11.71
(3)	$b_1\ln(H) + \underline{b_2}\ln(CL) + b_3\ln(BA)$	Reduction due to model (3) -reduction due to model (1)	Inconsistent signs in *CL* terms	10	6.89868	.68987	11.73
(4)	$b_1\ln(H) + b_2\ln(CL) + \underline{b_3}\ln(BA)$	Reduction due to model (4) -reduction due to model (1)	Inconsistent signs in *BA* terms	10	6.77222	.67722	11.51
(5)	$\underline{b_1}\ln(H) + \underline{b_2}\ln(CL) + b_3\ln(BA)$	Reduction due to model (5) -reduction due to model (2)	Inconsistent signs in *CL* terms	10	1.16358	.11636	1.98
(6)	$\underline{b_1}\ln(H) + b_2\ln(CL) + \underline{b_3}\ln(BA)$	Reduction due to model (6) -reduction due to model (2)	Inconsistent signs in *BA* terms	10	.81690	.08169	1.39
(7)	$b_1\ln(H) + \underline{b_2}\ln(CL) + \underline{b_3}\ln(BA)$	Reduction due to model (7) -reduction due to model (3)	Inconsistent signs in *BA* terms	10	1.07251	.10725	1.82
	complete model $\underline{b_1}\ln(H) + \underline{b_2}\ln(CL) + \underline{b_3}\ln(BA)$	Error	Inconsistent signs in *CL* and *BA* terms	177	10.41343	.05883	

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[1] Mean $\ln(CW) = 1.26648$ feet; standard deviation = 0.46956.
[2] Terms varying by species are underlined.
[3] Tabulated F values: $F^{10}_{177,0.05} = 1.63$.

Foliage Weight

The general model for predicting needle weight developed from 114 trees 3.5 inches (8.9 cm) d.b.h. and larger is:

$$\ln(WT) = b_0 + b_1\ln(D) + b_2\ln(H) + b_3\ln(CL) + b_4\ln(A) + b_5\ln(TPA) + b_6\ln(DREL)$$

The analysis of variance in table 8 is slightly different from that in tables 4 and 7. It was not possible to fit the complete model (species x parameter interactions for seven variables) in the REX regression program because of the dimension of the problem. Instead, the improvement in fit was analyzed between two models at a time by forming an F ratio in which the denominator is the mean square of the model with the most parameters in it. For instance, the general model was fit in (1). Then in (2) the intercept was varied by species, and the reduction in error between (1) and (2) associated with 9 d.f. was measured with an F-test. Since this reduction was significant, the species-intercept term was carried through in each successively more complex model, (3) through (8). Beyond fitting the intercept term

Table 8.—Analysis of improvement of ln(foliage weight)[1] regression models attainable by varying parameters by species, trees 3.5 inches d.b.h. and larger

Model[2]	Source	Remarks	Degrees of freedom	Marginal sum of squares	Mean square	F[3]
(1) $b_0 + b_1\ln(D) + b_2\ln(H) + b_3\ln(CL)$	Reduction due to model (1)		6	152.41829	25.40305	126.59
$+ b_4\ln(A) + b_5\ln(TPA) + b_6\ln(DREL)$	Residual from model (1)		107	21.47126	.20067	
(2) $\underline{b_{0_i}} + b_1\ln(D) + b_2\ln(H) + b_3\ln(CL)$	Reduction due to model (2)-(1)		9	8.43661	.93740	7.05
$+ b_4\ln(A) + b_5\ln(TPA) + b_6\ln(DREL)$	Residual from model (2)		98	13.03464	.13301	
(3) $\underline{b_{0_i}} + \underline{b_{1_i}}\ln(D) + b_2\ln(H) + b_3\ln(CL)$	Reduction due to models (3)-(2)		9	1.35027	.15003	1.14
$+ b_4\ln(A) + b_5\ln(TPA) + b_6\ln(DREL)$	Residual from model (3)		89	11.68435	.13128	
(4) $\underline{b_{0_i}} + b_1\ln(D) + \underline{b_{2_i}}\ln(H) + b_3\ln(CL)$	Reduction to models (4)-(2)		9	1.60567	.17841	1.39
$+ b_4\ln(A) + b_5\ln(TPA) + b_6\ln(DREL)$	Residual from model (4)		89	11.42897	.12842	
(5) $\underline{b_{0_i}} + b_1\ln(D) + b_2\ln(H) + \underline{b_{3_i}}\ln(CL)$	Reduction due to model (5)-(2)		9	2.20900	.24544	2.02
$+ b_4\ln(A) + b_5\ln(TPA) + b_6\ln(DREL)$	Residual from model (5)	inconsistent signs in CL terms	89	10.82565	.12164	
(6) $\underline{b_{0_i}} + b_1\ln(D) + b_2\ln(H) + b_3\ln(CL)$	Reduction due to models (6)-(2)		9	.75875	.08431	.61
$+ \underline{b_{4_i}}\ln(A) + b_5\ln(TPA) + b_6\ln(DREL)$	Residual due to model (6)	inconsistent signs in A term	89	12.27590	.13793	
(7) $\underline{b_{0_i}} + b_1\ln(D) + b_2\ln(H) + b_3\ln(CL)$	Reduction due to models (7)-(2)		9	.81401	.09446	.66
$+ b_4\ln(A) + \underline{b_{5_i}}\ln(TPA) + b_6\ln(DREL)$	Residual from model (7)	inconsistent signs in TPA term	89	12.22063	.13731	
(8) $\underline{b_{0_i}} + b_1\ln(D) + b_2\ln(H) + b_3\ln(CL)$	Reduction due to models (8)-(2)		9	1.41581	.15731	1.20
$+ b_4\ln(A) + b_5\ln(TPA) + \underline{b_{6_i}}\ln(DREL)$	Residual from model (8)	inconsistent signs in DREL terms	89	11.61885	.13055	

[1] Mean in(WT) = 3.41782 lb; standard deviation = 1.24050.
[2] Terms varying by species are underlined.
[3] Tabulated F values: $F^{9}_{98,0.01} = 2.59$; $F^{9}_{89,0.05} = 1.98$

by species, there was no significant species interaction with any of the variables in the general model except for ln(CL). However, since the signs of the coefficients for the species x ln(CL) term were inconsistent, this model was rejected as possibly overfitting the data.

Thus, the final model is that in which just the intercept term varies by species:

$$\ln(WT) = b_{0_i} + b_1\ln(D) + b_2\ln(H) + b_3\ln(CL) + b_4\ln(A) + b_5\ln(TPA) + b_6\ln(DREL)$$

The coefficients in table 9 gave a residual mean square of 0.13301. Note that since there were no data available for lodgepole pine, no parameters were estimated for this species. Table 10 shows for which species the model overpredicts and underpredicts. The mean prediction underestimates foliage biomass by 86.9 lb (39.4 kg). A correction for bias of $e^{1\ 2(\ 13301)} = 1.069$ may be applied, resulting in an overprediction of 5.7 lb (2.6 kg) per tree after adjustment.

In the above model, predicted foliage weight decreases as trees per acre increase, an indication of competitive stress due to stocking. The positive crown length and negative height, which reflect crown ratio in the natural scale, and positive relative diameter terms may all be interpreted as measures of an individual tree's competitive status. In addition, the negative age term coupled with the positive diameter term may be interpreted as an estimator of mean annual diameter increment in the natural scale, which would be expected to vary directly with foliage weight. As tree age is not always a readily available measurement in inventory data, an alternative equation for foliage weight involving a transformation of the diameter and age terms is presented in a later section.

Table 9.—Coefficients for estimating ln(foliage weight) of trees 3.5 inches and larger:
$\ln(WT) = b_{0_i} + b_1\ln(D) + b_2\ln(H) + b_3\ln(CL) + b_4\ln(A) + b_5\ln(TPA) + b_6\ln(DREL)$

Species[1]	Variable coefficients Intercept b_{0_i}
BP	2.62251
WP	2.66607
WC	3.05935
WL	1.75654
WH	2.65457
AF	3.06017
GF	3.11508
ES	3.30085
DF	2.70587
PP	2.45249
Variables	**Variable coefficients**
ln(D)	$b_1 = 2.08624$
ln(H)	$b_2 = -1.07705$
ln(CL)	$b_3 = .69082$
ln(A)	$b_4 = -.30885$
ln(TPA)	$b_5 = -.14210$
ln(DREL)	$b_6 = .39924$
Regression sum of squares/d.f.	= 160.85490/15
Error sum of squares/d.f.	= 13.03465/98
Mean square error	= 0.13301
R^2	= 0.9250

[1] See text page 2 for species list.

In developing an equation for foliage weight prediction for 210 trees less than 3.5 inches (8.9 cm) d.b.h., the general model form is:

$$\ln(WT) + b_0 + b_1\ln(CL) + b_2\ln(A) + b_3\ln(TPA)$$

The analysis of variance of species by parameter interactions for this model is shown in table 11. Varying the intercept and ln(*CL*) by species resulted in significant improvement while maintaining consistency in signs between species. Improvements due to varying other parameters by species were also significant, but integrity of sign was no longer maintained. Therefore, the model chosen for predicting foliage weight of trees less than 3.5 inches (8.9 cm) is:

$$\ln(WT) = b_{0_i} + b_{1_i}\ln(CL) + b_2\ln(A) + b_3\ln(TPA)$$

where the subscript *i* denotes a parameter varying by species.

Table 10.—Summary of residuals (natural scale) from prediction equations for foliage weight of trees. Bias adjustment factors are 1.069 for trees greater than or equal to 3.5 inches, and 1.126 for trees less than 3.5 inches

	Trees ≥3.5 inches d.b.h.			Trees <3.5 inches d.b.h.		
		Σ (obs-pred)			Σ (obs-pred)	
Species[1]	No. of trees	without correction	with bias correction	No. of trees	without correction	with bias correction
BP	4	0.44	−3.92	12	1.31	−0.07
WP	2	−2.77	−4.65	17	1.85	−1.90
WC	22	−218.20	−330.13	25	5.77	.55
WL	3	−.05	−.98	15	−.16	−1.90
WH	4	1.51	−5.23	16	1.34	−2.96
AF	10	−2.22	−30.10	15	5.51	.28
LP	0	—	—	12	0.01	−1.26
GF	17	48.58	−4.29	29	3.88	−5.35
ES	3	4.16	−5.42	14	1.56	−3.13
DF	20	−52.94	−172.73	28	7.61	1.43
PP	29	515.50	362.32	27	5.15	1.10
Weighted mean residual (lbs/tree)		86.89	−5.69		3.71	−1.12
(kg/tree)		(39.41)	(−2.58)		(1.68)	(−0.51)

[1] See text page 2 for species list.

Table 11.—Analysis of improvement of ln(foliage weight)[1] regression models attainable by varying parameters by species, trees less than 3.5 inches d.b.h.

	Model[2]	Source	Remarks	Degrees of freedom	Marginal sum of squares	Mean square reduction	F[3]
(1)	$b_0 + b_1 \ln(CL) + b_2 \ln(A) + b_3 \ln(TPA)$	Reduction due to model (1)		3	259.86645	86.62215	413.06
(2)	$\underline{b_{0_j}} + b_1 \ln(CL) + b_2 \ln(A) + b_3 \ln(TPA)$	Reduction due to model (2) -reduction due to model (1)		10	32.33326	3.23326	15.42
(3)	$\underline{b_{0_j}} + \underline{b_{1_j}} \ln(CL) + b_2 \ln(A) + b_3 \ln(TPA)$	Reduction due to model (3) -reduction due to model (2)		10	5.37841	.53784	2.56
(4)	$\underline{b_{0_j}} + b_1 \ln(CL) + \underline{b_{2_j}} \ln(A) + b_3 \ln(TPA)$	Reduction due to model (4) -reduction due to model (2)	inconsistent signs in *A* term	10	6.87378	.68738	3.28
(5)	$\underline{b_{0_j}} + b_1 \ln(CL) + b_2 \ln(A) + \underline{b_{3_j}} \ln(TPA)$	Reduction due to model (5) -reduction due to model (2)	inconsistent signs in *TPA* term	10	5.20996	.52100	2.48
(6)	$\underline{b_{0_j}} + \underline{b_{1_j}} \ln(CL) + \underline{b_{2_j}} \ln(A) + b_3 \ln(TPA)$	Reduction due to model (6) -reduction due to model (3)	inconsistent signs in *A* term	10	5.54004	.55400	2.64
(7)	$\underline{b_{0_j}} + \underline{b_{1_j}} \ln(CL) + b_2 \ln(A) + \underline{b_{3_j}} \ln(TPA)$	Reduction due to model (7) -reduction due to model (3)	inconsistent signs in *TPA* terms	10	5.38501	.53850	2.57
(8)	$\underline{b_{0_j}} + b_1 \ln(CL) + \underline{b_{2_j}} \ln(A) + \underline{b_{3_j}} \ln(TPA)$	Reduction due to model (8) -reduction due to model (4)	inconsistent signs in *A* and *TPA* terms	10	3.79663	.37966	1.81
	complete model $\underline{b_{0_j}} + \underline{b_{1_j}} \ln(CL) + \underline{b_{2_j}} \ln(A) + \underline{b_{3_j}} \ln(TPA)$	Error	inconsistent signs in *A* and *TPA* terms	166	34.81201	.20971	

[1] Mean $\ln(WT) = -0.075104$ lb; standard deviation = 1.27874.

[2] Terms varying by species are underlined.

[3] Tabulated F values: $F^{10}_{166\ 0.05} = 1.89$; $F^{10}_{166.0\ 01} = 2.43$.

Mean square error for the above model is 0.23751, and coefficient values are shown in table 12. Table 10 summarizes the residuals for the model. Foliage weight is underestimated by an average of 3.7 lb (1.7 kg) per tree. Negative bias may be compensated for by multiplying the estimate in the natural scale by $e^{1/2(.23751)} = 1.126$, giving an overprediction of 1.1 lb (0.5 kg) per tree after adjustment.

The present approach for estimating foliage biomass uses two model forms; one for small trees and one for large trees — each of which is applicable to all species. The data used in developing these equations were a subset of Brown's (1978) data. His approach was to fit one or more separate model forms for total crown biomass (foliage and branchwood) for trees greater than 1 inch d.b.h. and for trees less than 2 inches d.b.h., by species and crown class. He further fitted separate regression equations for predicting the proportion of total crown biomass in foliage and branchwood components. Foliage weight can be estimated indirectly using Brown's method by applying these two equations in succession to the data. The following method was used to compare the present approach with Brown's approach for predicting foliage biomass. The best combination (that yielding lowest mean square error) of Brown's crown biomass and foliage proportion equations for a species was solved for each tree in the data set used to develop the current equations. Residual sums of squares [Σ(observed − predicted needle weight)^2] were then compared for the two methods. Results are as follows:

Table 12.—Coefficients for estimating ln (foliage weight) of trees less than 3.5 inches d.b.h.:
$\ln(WT) = b_{0_i} + b_{1_i}\ln(CL) + b_2\ln(A) + b_3\ln(TPA)$

Species[1]	Variable coefficients Intercept b_{0_i}	$\ln(CL)$ b_{1_i}
BP	−2.81317	1.47513
WP	−2.15894	1.48969
WC	−2.64034	1.69973
WL	−5.02156	2.31835
WH	−4.22701	2.22534
AF	−2.03919	1.64942
LP	−3.38394	1.96060
GF	−2.78090	1.90272
ES	−3.30673	2.27613
DF	−2.30430	1.52896
PP	−3.02050	1.88712
Variables	Variable coefficients	
$\ln(A)$	$b_2 = .22823$	
$\ln(TPA)$	$b_3 = -.13550$	

Regression sum of squares d.f = 297.57812 23
Error sum of squares d.f. = 44.17596 186
Mean square error = 0.23751
R^2 = 0.8707

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[1] See text page 2 for species list.

Residual sums of squares for prediction of ln (foliage weight)

	Trees <3.5 *inches* d.b.h.	Trees ≥3.5 *inches* d.b.h.	Total
Current models	44.17596	13.0346	57.21056
Brown's models	69.9460	10.08098	80.0269

The current model for trees greater than or equal to 3.5 inches (8.9 cm) d.b.h. had a larger residual sum of squares than that obtained from Brown's methods. For trees less than 3.5 inches (8.9 cm), residual sum of squares obtained by the present method are lower. Overall there appears to be no loss in predictive ability by using a single model form as presented here.

Alternative Models for Foliage Weight Prediction

The prediction models for both large and small trees use the variable ln(age), which is not available in the prognosis system (Stage 1973) in which the equations are to be used. Thus the following alternative models are presented.

For trees 3.5 inches (8.9 cm) and larger, the alternative model is a transformation of the general model involving the diameter and age terms. Periodic (diameter)^2 increment, which is available in the prognosis model, is substituted for mean annual (diameter)^2 increment in the following transformation:

$$\ln(WT) = b_{0_i} + b_1\ln(D) + b_2\ln(H) + b_3\ln(CL) + b_4\ln(A) + b_5\ln(TPA) + b_6\ln(DREL)$$
(general model in log linear form)

or,

$$WT = e^{b_{0_i}} \cdot D^{b_1} \cdot H^{b_2} \cdot (CL)^{b_3} \cdot A^{b_4} \cdot (TPA)^{b_5} \cdot (DREL)^{b_6}$$
(general model in exponential form)

Multiplying by $\frac{D^{2b_4}}{D^{2b_4}}$

$$WT = e^{b_{0_i}} \cdot D^{(b_1 + 2b_4)} \left(\frac{D^2}{A}\right)^{-b_4} \cdot H^{b_2} \cdot (CL)^{b_3} \cdot (TPA)^{b_5} \cdot (DREL)^{b_6}$$

which, in logarithmic form, is

$$\ln(WT) = b_{0_i} + (b_1 + 2b_4)\cdot\ln(D) - b_4\ln\left(\frac{D^2}{A}\right) + b_2\ln(H) + b_3\ln(CL) + b_5\ln(TPA) + b_6\ln(DREL)$$

and substituting periodic $[\Delta(D^2)]$ for mean annual $[D^2/A)]$ increment,

$$\ln(WT) = b_{0_i} + (b_1 + 2b_4)\ln(D) - b_4\ln[\Delta(D^2)] + b_2\ln(H) + b_3\ln(CL) + b_5\ln(TPA) + b_6\ln(DREL)$$
(alternative model)

Kittredge (1944) noted that it would be expected that "the amount of foliage could be estimated from the periodic annual growth because the increment of stem wood is determined by the amount of foliage which is carrying on photosynthesis in that period" (p. 906). It is important to note that the alternative model has not been reparameterized with the periodic annual increment (PAI) as an independent variable since PAI was not measured. It is merely a transformation of the original parameters and should be presented with a statement of caution about the relationship between the mean annual increment (MAI) and PAI.

The substitution of periodic (diameter)^2 increment for mean annual increment in the alternative equation assumes that the relationship between (diameter)^2 and age is linear over a wide range of ages. The assumption breaks down when PAI and MAI diverge as typically happens over time except at the culmination of MAI. Figure 1 shows the difference in predicted foliage weight between the current and alternative models that can be expected when PAI departs from MAI. When PAI is 100 percent greater than MAI, predicted foliage biomass increases 24 percent. PAI 60 percent of MAI produces a 25 percent negative change in predicted foliage biomass.

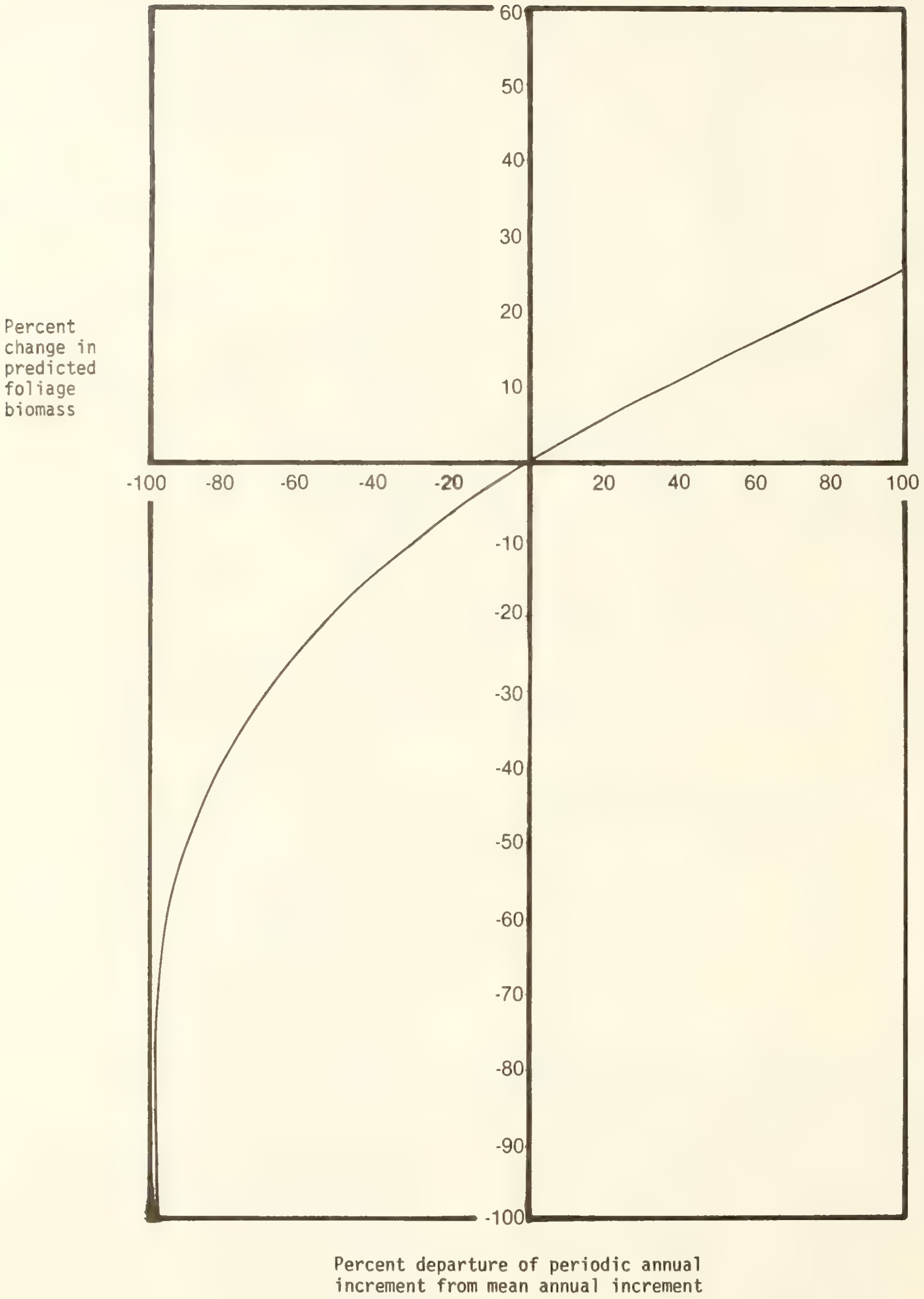

Figure 1.—Change in predicted foliage biomass as periodic annual increment departs from mean annual increment.

To compare whether the alternative model approximates the fit obtained by the general model, both models were applied to an independent stand data set[4] in which both periodic and mean annual increments were measured. The tree ages in this stand ranged from 2 to 560 years. For 95 trees in the test data set, the chi-square value for the sum of squared differences between needle weight prediction by the alternative model and prediction by the general model was 4.10 compared to the tabulated value of $\chi^2_{95,\ 0.005} = 134.24$. The alternative model overpredicts needle weight outside of the range of data used to develop the general model (the sum of differences between predictions by the alternative and current models was 741.3 lb [336.2 kg] for the 95 trees). However, with the low chi-square value, the alternative model may be judged adequate, and also uses independent variables that are more readily available.

For trees less than 3.5 inches (8.9 cm) d.b.h., the following reparameterized alternative model substituting height for age is presented:

$$\ln(WT) = b_{0_i} + b_1\ln(H) + b_{2_i}\ln(CL) + b_3\ln(TPA)$$

Mean square error is 0.24759 for the coefficients given in table 13. This model underestimates needle weight for 9 of 11 species (table 14). On the average, it underpredicts needle weight by 3.6 lb (1.6 kg). A correction for underestimation of $e^{1/2(.24759)} = 1.132$ may be applied giving a mean overprediction of 1.0 lb (0.5 kg) after bias adjustment. Residual sum of squares obtained for this alternative equation is 46.0511, which is also lower than that obtained from Brown's method. Thus, the alternative formulation is judged acceptable.

Table 13.—Coefficients for estimating ln (foliage weight) of trees less than 3.5 inches d.b.h., alternative model: $\ln(WT) = b_{0_i} + b_1\ln(H) + b_{2_i}\ln(CL) + b_3\ln(TPA)$

Species[1]	Variable coefficients Intercept b_{0_i}	 ln(*CL*) b_{2_i}
BP	−2.63387	1.35092
WP	−1.94951	1.22023
WC	−2.24876	1.37600
WL	−4.73762	1.98479
WH	−4.17456	2.00749
AF	−1.60998	1.32649
LP	−3.13488	1.62368
GF	−2.43200	1.60270
ES	−2.93508	1.96125
DF	−2.05828	1.25837
PP	−2.74410	1.58171
Variables	Variable coefficients	
ln(*H*)	$b_1 = .40350$	
ln(*TPA*)	$b_2 = -.12975$	

Regression sum of squares/d.f. = 295.70288 23
Error sum of squares/d.f. = 46.05106 186
Mean square error = 0.24759
R^2 = 0.8653

[1] See text page 2 for species list.

Table 14.—Summary of residuals (natural scale) from alternative prediction equation for foliage weight of trees less than 3.5 inches d.b.h. The bias correction factor is 1.132

Species[1]	No. of trees	Σ (obs — pred) without bias correction	 with bias correction
BP	12	1.41	0.11
WP	17	1.85	−1.74
WC	25	5.74	.74
WL	15	−.45	−2.15
WH	16	1.14	−3.00
AF	5	5.83	.87
LP	12	−.09	−1.32
GF	29	3.44	−5.29
ES	14	1.63	−2.85
DF	28	7.39	1.46
PP	27	5.23	1.36
Weighted mean residual			
(lb/tree)		3.62	−1.03
(kg/tree)		(1.64)	(−.47)

[1] See text page 2 for species list.

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[4] St. Joe National Forest, Idaho.

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Relationships between tree crown width and diameter at breast height, height, crown length, and basal area per acre, and between tree foliage biomass and diameter at breast height, height, crown length, age, relative diameter, and trees per acre are presented for 11 conifer species in the Northern Rocky Mountains.

KEYWORDS: crown width, foliage biomass

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